Threshold Corrections and Gauge Symmetry in Twisted Superstring Models

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Threshold corrections to the running of gauge couplings are calculated for superstring models with free complex world sheet fermions. For two N=1 $SU(2) \times U(1)^5$ models, the threshold corrections lead to a small increase in the unification scale. Examples are given to illustrate how a given particle spectrum can be described by models with different boundary conditions on the internal fermions. We also discuss how complex twisted fermions can enhance the symmetry group of an N=4 $SU(3) \times U(1) \times U(1)$ model to the gauge group $SU(3) \times SU(2) \times U(1)$. It is then shown how a mixing angle analogous to the Weinberg angle depends on the boundary conditions of the internal fermions.

1. Introduction

The unification of gauge coupling constants is a necessary consequence of string theory. At tree level, the gauge couplings have simple relations to the string coupling constant. In

higher orders of perturbation theory, this relation holds only at the Planck mass. Below this energy, the gauge couplings evolve as determined by the renormalization group equations. Threshold effects [1] can also modify the tree level relation and shift the unification scale. Although the effect of thresholds is small in grand unified field theories, the threshold corrections can be quite large in string theories [2-7] because there is an infinite tower of massive states, all of which contribute.

In this paper, we calculate threshold corrections in four-dimensional critical superstring models written in terms of free fermions with twisted boundary conditions. Complex fermions are useful for studying theories with chiral space-time fermions and for probing the structure of gauge symmetries. Here, we adapt the background field method [5,8] for calculating string thresholds to twisted models in the framework of type II theories. Although a phenomenologically realistic type II model has not yet been found, it provides a more economical construction for using the techniques of low energy string phenomenology. Calculations of thresholds in heterotic models[5-13] can be made large enough to lower the unification scale to an acceptable energy. This is achieved by fine tuning the many free moduli parameters of the theory. A desirable feature of type II strings is that there is less freedom to adjust the parameters of the theory.

In sect. 2, we give a general discussion of the background field method for running couplings in twisted models, emphasizing models with higher level Kac-Moody currents. In sect. 3, we calculate the threshold corrections for two twisted chiral models with $SU(2) \times U(1)^5$ gauge symmetry. These two models have the same massless particle spectrum but

different boundary conditions on the internal fermions. This shows how different boundary conditions on the internal fermions affect the thresholds. In sect. 4, we investigate the relationship between the boundary conditions on the internal fermions and the ratio of the field theory couplings at the Plank mass[14]. Specifically, it will be shown how the twisted boundary conditions can enhance the symmetry group of an N=4 $SU(3) \times U(1) \times U(1)$ model to $SU(3) \times SU(2) \times U(1)$. We then determine a mixing angle analogous to the Weinberg angle of the standard model.

2. Background field calculation

The tree level relation between gauge couplings is [15]:

$$\frac{4\pi}{g_a^2} = 2x_a \frac{4\pi}{g_{str}^2} \tag{2.1}$$

where x_a is the level of Kac-Moody Algebra = N for SU(N). The factor of 2 is present because we choose a field theory normalization for the longest roots equal to 1. This relation is determined by comparing the scattering amplitudes (e.g. three-point gauge boson vertex) for the low energy string theory (massless modes) to field theory. The tree amplitudes are identical if one makes this identification, which holds at the Planck mass. We now want to calculate how this relation changes when higher order corrections and threshold effects are considered. This involves using the background field method [5,8,17] to find the threshold corrections. Since we are interested in models with chiral space-time fermions, we consider models with complex twisted fermions.

The background field method involves describing the effective action of the quantum gauge field as an effective action of a string propagating in a classical background gauge field:

$$\Gamma[A^a_\mu] = \Gamma[X^\mu = 0, \Psi^\mu = 0, A^a_\mu]\,. \tag{2.2}$$

The effective action will now include contributions from the massive modes of the string. X^μ, $\Psi^\mu = 0$ means that there are no external string states. In addition, since the gauge fields are classical, they do not circulate in loops. In other words, the classical gauge fields only exist as external states. Polchinski[16] derived a formula for the one-loop correction to (2.1):

$$\Gamma[X^\mu = 0, \Psi^\mu = 0, A^a_\mu] = \int d^4x(-\frac{1}{4g_a^2}F^a_{\mu\nu}F^{a\mu\nu}) + \int \frac{d^2\tau}{\tau_2}\mathbf{Z} + ... \tag{2.3}$$

where $\mathbf{Z}$ is the partition function (one loop with no external states) of the string in the presence of a background gauge field. $\tau = \tau_1 + i\tau_2$ are the coordinates on the torus. The first term in (2.3) is simply the classical action of the gauge field and $-\frac{1}{4g_a^2}$ is the tree approximation for the gauge coupling. The second term, which is the one-loop correction to the tree level result, can be shown to be equivalent to a one-loop two point string amplitude where the string vertex operator for the emission of a gauge boson is modified by making the substitution $\epsilon_\mu e^{ik\cdot X(z,\bar{z})} \rightarrow A_\mu = -\frac{1}{2}F_{\mu\nu}X^\nu$ provided that A_μ satisfies the classical equations of motion. The first order correction to the field theory coupling constants is then given by the the coefficient of the $-\frac{1}{4}F^2_{\mu\nu}$term in the one-loop two point

background gauge field amplitude. We now outline this method for models containing twisted fermions.

Type II 4-dimensional string models can contain chiral space-time fermions [18-21] if some of the internal coordinates take values on a shifted lattice $\sqrt{2\alpha'}p \in Z+\nu$. These complex twisted fermions satisfy the following boundary conditions:

$$\psi(e^{2\pi i}z) = -e^{2\pi i\nu}\psi(z) \quad ; \quad \tilde{\psi}(e^{2\pi i}z) = -e^{-2\pi i\nu}\tilde{\psi}(z) \tag{2.4}$$

where ν is real. The space-time fermions are real, either Neveu-Schwarz or Ramond. They are given by:

$$\psi^\mu(z) = \sum_{r\in Z+\frac{1}{2}} \psi^\mu_r z^{-r-\frac{1}{2}} \quad ; \qquad \psi^\mu(z) = \sum_{n\in Z} \psi^\mu_n z^{-n-\frac{1}{2}} \tag{2.5}$$

for Neveu-Schwarz and Ramond respectively. Twisted models differ from untwisted models in that internal fermions can be defined by the following for any value of λ

$$\psi^i(z) = \sum_{r\in Z+\lambda} \psi^i_r z^{-r-\frac{1}{2}} \quad ; \quad \tilde{\psi}^i(z) = \sum_{r\in Z-\lambda} \tilde{\psi}^i_r z^{-r-\frac{1}{2}} \tag{2.6}$$

where $\lambda = \frac{1}{2} - \nu$ and $f^\dagger_r = \tilde{f}_{-r}$. $\nu = 1/2$ is the Ramond case and $\nu = 0$ the Neveu-Schwarz case.

We consider 4-dimensional type II models in the light-cone description. Here, the left and right movers can each be described by two bosonic and twenty fermionic fields. The

partition function without the zero modes p is given by

$$
\begin{aligned}
Z &= \prod_{l=0}^{k-1} \frac{1}{\mathcal{N}_l} \sum_{\alpha,\beta} c(\alpha,\beta) Tr_\alpha [q^{L_0'-\frac{1}{2}} \bar{q}^{\bar{L}_0'-\frac{1}{2}} (e^{-i\pi})^{\rho_\beta \cdot F}] \\
&= \prod_l \frac{1}{\mathcal{N}_l} \sum_{\alpha,\beta} c(\alpha,\beta) |\eta(q)|^{-24} \prod_{i=1}^{n} \left(\vartheta \begin{bmatrix} \rho_\alpha^i \\ \rho_\beta^i \end{bmatrix} (0|q) \right)^{1/2} \prod_{i=1}^{n'} \left(\vartheta \begin{bmatrix} \bar{\rho}_\alpha^i \\ \bar{\rho}_\beta^i \end{bmatrix} (0|\bar{q}) \right)^{1/2} \\
&\times \prod_{i=1}^{m} \vartheta \begin{bmatrix} \rho_\alpha^i \\ \rho_\beta^i \end{bmatrix} (0|q) \prod_{i=1}^{m'} \bar{\vartheta} \begin{bmatrix} \rho_\alpha^i \\ \rho_\beta^i \end{bmatrix} (0|\bar{q})
\end{aligned}
\tag{2.7}
$$

where the prime on the Hamiltonian denotes the omission of the bosonic zero modes. In this formalism, the partition function is a sum over the sectors generated by $\rho_\alpha \equiv (\rho_\alpha; \bar{\rho}_\alpha) \in \Omega$. Each sector α contains $n+n'$ real fermions and $m+m'$ complex fermions. ρ_α is a $(n+n'+m+m')$ dimensional vector which describes the boundary conditions of the fermions for each sector:

$$
\rho_\alpha = 2(\nu_1, ...\nu_n; \nu_1, ...\nu_m; \nu_1, ...\nu_{n'}; \nu_1, ...\nu_{m'}) \,. \tag{2.8}
$$

$c(\alpha,\beta) = \delta_\alpha \epsilon(\alpha,\beta)$ are phases for the $(e^{-i\pi})^{\rho_\beta \cdot F}$ projections. F is a vector whose components are the operators $F_j = \sum_{r \in z+\lambda} : f_r^j \tilde{f}_{-r}^j :$ for complex fermions and $\sum_{s=1/2}^\infty b_{-s}^j b_s^j$ or $\sum_1^\infty d_{-n}^j d_n^j$ for real NS or R fermions respectively.

$$
\rho_\beta \cdot F = 2\sum_{j=1}^{n} \nu_j F_j^L + 2\sum_{j=1}^{m} \nu_j F_j^L - 2\sum_{j=1}^{n\prime} \nu_j F_j^R - 2\sum_{j=1}^{m\prime} \nu_j F_j^R \,. \tag{2.9}
$$

In addition, the factor $\prod_{l=0}^{k-1} \mathcal{N}_l$ (k= the number of generators) is the number of sectors where the order, $\mathcal{N}_\alpha$ is defined by $\alpha^{\mathcal{N}_\alpha} = \phi = ((1)^n; (1)^m; (1)^{n'}; (1)^{m'})$ and α is a vector whose components are given by

$$
\alpha = (e^{2\pi i \nu}, ...; e^{2\pi i \nu}, ...; e^{2\pi i \nu}, ...; e^{2\pi i \nu}, ...) \tag{2.10}
$$

The generalized Jacobi theta functions are given by

$$\vartheta \begin{bmatrix} \rho \\ \mu \end{bmatrix} (\nu|\tau) = \sum_{n\in Z} e^{i\pi\tau(n+\rho/2)^2} e^{-i2\pi(n+\rho/2)(\nu+\mu/2)} e^{i\pi\rho\mu/2}$$

$$\bar{\vartheta} \begin{bmatrix} \bar{\rho} \\ \bar{\mu} \end{bmatrix} (\nu|\bar{\tau}) = \sum_{n\in Z} e^{-i\pi\bar{\tau}(n+\bar{\rho}/2)^2} e^{i2\pi(n+\bar{\rho}/2)(\nu+\bar{\mu}/2)} e^{-i\pi\bar{\rho}\bar{\mu}/2} \tag{2.11}$$

with $q = e^{2\pi i\tau}$, $\bar{q} = e^{-2\pi i\bar{\tau}}$,$\tau = \tau_1 + i\tau_2$, and $\bar{\tau} = \tau_1 - i\tau_2$. Note that this differs from that given in [18] because the left movers are now functions of q rather that $\bar{q}$.

The one-loop two point amplitude contribution to the effective lagrangian for A^a_μ background is :

$$\mathcal{L}'(A^a_\mu) = \prod_l \frac{1}{\mathcal{N}_l} \sum_{\alpha,\beta} c(\alpha,\beta) \int \frac{d^4p}{(2\pi)^4} Tr_\alpha[\Delta V^a(1,1)\Delta V^a(1,1)(e^{-i\pi})^{\rho_\beta\cdot F}] \tag{2.12}$$

where the sum over sectors corresponds to a generalized GSO projection[18]. The closed string propagator is

$$\Delta = \frac{1}{4\pi} \int_{|z|\leq 1} \frac{dzd\bar{z}}{|z|^2} z^{L_0 - \frac{1}{2}} \bar{z}^{\bar{L}_0 - \frac{1}{2}} \, . \tag{2.13}$$

The Hamiltonian L_0 and associated Virasoro generators are given by:

$$L_n = \sum_{r\in Z+\lambda} (r - \frac{n}{2}) : \tilde{f}_{n-r} f_r : + \frac{1}{4} \sum (\lambda - \frac{1}{2})^2 \delta_{n.0} \, . \tag{2.14}$$

Recall that the background field method involves making the substitution $\epsilon_\mu e^{ik\cdot X(z,\bar{z})} \rightarrow A_\mu(x)$ in the vertex operator for a gauge boson provided that $A_\mu(x)$ satisfies the equation of motion $\partial_\mu F^{\mu\nu} = 0$. The vertex operator for a gauge boson, $b^{La}_{-\frac{1}{2}} \epsilon \cdot b^R_{-\frac{1}{2}} |0>$, is constructed in

part from the Kac-Moody currents of (2.15) and (2.16). In models with complex fermions, such as the two chiral models that are discussed in sect. 3, the affine algebra is constructed for a particular gauge group. For a model with complex fermions and an $SU(2) \times U(1)^5$ gauge symmetry, the currents are given by:

$$J^a(z) = \sum_{n \in Z} J^a_n z^{-z} = -\frac{i}{2} f_{abc} \psi^b(z) \psi^c(z) \tag{2.15}$$

where f_{abc} are the structure constants of $SU(2)$ and $3 \leq a, b, c \leq 5$.

The U(1) currents are:

$$J^a(z) =: f^j(z) \tilde{f}^j(z) : + \nu_j \tag{2.16}$$

for $6 \leq a \leq 10$, $1 \leq j \leq 5$, and no sum on j. The zero modes of these currents generate the gauge symmetry. The vertex operator for an $SU(2)$ gauge boson is:

$$\begin{aligned} V^a(k, \epsilon, z, \bar{z}) =& [\tfrac{1}{2} k \cdot \psi^L(z) \psi^{La}(z) - \tfrac{i}{2} f_{abc} \psi^{Lb}(z) \psi^{Lc}(z)] \\ & \epsilon \cdot [i \bar{z} \bar{\partial} X_{\mathrm{R}}(\bar{z}) - \tfrac{1}{2} \psi^R(\bar{z}) k \cdot \psi^R(\bar{z})] e^{ik \cdot X(z, \bar{z})} \end{aligned} \tag{2.17}$$

Here, all the fermionic oscillators are real. On the other hand, the vertex operators for the U(1) gauge bosons are constructed from the corresponding Kac-Moody currents :

$$\begin{aligned} V^a(k, \epsilon, z, \bar{z}) =& [\tfrac{1}{2} k \cdot \psi^L(z) \psi^{La}(z) + : \psi^{Lj}(z) \tilde{\psi}^{Lj}(z) :] \\ & \epsilon \cdot [i \bar{z} \bar{\partial} X_{\mathrm{R}}(\bar{z}) - \tfrac{1}{2} \psi^R(\bar{z}) k \cdot \psi^R(\bar{z})] e^{ik \cdot X(z, \bar{z})} \end{aligned} \tag{2.18}$$

where ν_j in the current from (2.16) is zero. The vertex operator also contains the bosonic fields given by

$$\begin{aligned} X^\mu(z, \bar{z}) &= x^\mu + \frac{p^\mu}{4i} (\ln z + \ln \bar{z}) + \frac{i}{2} \sum_{n \neq 0} \frac{1}{n} \tilde{\alpha}^\mu_n z^{-n} + \frac{i}{2} \sum_{n \neq 0} \frac{1}{n} \alpha^\mu_n \bar{z}^{-n} \\ &= \frac{1}{2} \left(X^\mu_{\mathrm{L}}(z) + X^\mu_{\mathrm{R}}(\bar{z}) \right) \end{aligned} \tag{2.19}$$

It is only necessary to evaluate one component of a non-abelian subgroup at a time. For example, for $SU(2)$, one would look at only one of the three gauge bosons . Due to gauge invariance, it doesn't matter which one is selected. For a constant $F_{\mu\nu}$ corresponding to a given component of subgroup a, the resulting background field vertex is

$$\begin{aligned} V^a[F_{\mu\nu}](z,\bar{z}) = \tfrac{i}{4}F_{\mu\nu}\{J^a(z)[2X^\mu(z,\bar{z})\bar{z}\bar{\partial}X^\nu_{\mathrm{R}}(\bar{z}) - \psi^{R\mu}(\bar{z})\psi^{R\nu}(\bar{z})] \\ - i[\psi^{L\mu}(z)\psi^{La}(z)][\bar{z}\bar{\partial}X^\nu_{\mathrm{R}}(\bar{z})]\} \end{aligned} \tag{2.20}$$

where the gauge currents are given in (2.15) and (2.16). Using operator methods one can rewrite (2.12) as:

$$\begin{aligned} \mathcal{L}'(A^a_\mu) = \prod_l \frac{1}{\mathcal{N}_l}\sum_{\alpha,\beta} c(\alpha,\beta)\pi^2 \int \frac{d^4p}{(2\pi)^4}\int_\Gamma d^2\tau \int_{0\le Im\nu\le Im\tau} d^2\nu \\ \times Tr_\alpha[V^a(z,\bar{z})V^a(1,1)q^{L_0-1/2}\bar{q}^{\bar{L}_0-1/2}(e^{-i\pi})^{\rho_\beta\cdot F}] \end{aligned} \tag{2.21}$$

where z=$e^{2\pi i\nu}$, $q = e^{2\pi i\tau}$, and the integration is restricted to the fundamental region Γ. Performing the trace:

$$\begin{aligned} & Tr_\alpha[V^a(z,\bar{z})V^a(1,1)q^{L_0-1/2}\bar{q}^{\bar{L}_0-1/2}(e^{-i\pi})^{\rho_\beta\cdot F}] = \\ & \quad - \tfrac{1}{16}F_{\mu\nu}F_{\rho\sigma}Tr_\alpha[q^{L_0-1/2}\bar{q}^{\bar{L}_0-1/2}(e^{-i\pi})^{\rho_\beta\cdot F}] \\ & \quad \times \{\langle J^a(z)J^a(1)\rangle[\langle 2X^\mu(z,\bar{z})\bar{z}\bar{\partial}X^{R\nu}(\bar{z})2X^\rho(1,1)\bar{\partial}X^{R\sigma}(1)\rangle + \langle\psi^\mu(\bar{z})\psi^\nu(\bar{z})\psi^\rho(1)\psi^\sigma(1)\rangle] \\ & \quad - \langle\psi^{L\mu}(z)\psi^{La}(z)\psi^{L\rho}(1)\psi^{La}(1)\rangle\langle\bar{z}\bar{\partial}X^{R\nu}(\bar{z})\bar{\partial}X^{R\sigma}(1)\rangle\} \end{aligned} \tag{2.22}$$

where the two point correlation function on a torus is defined by:

$$\langle A(z,\bar{z})B(w,\bar{w})\rangle \equiv \frac{Tr_\alpha[A(z,\bar{z})B(w,\bar{w})q^{L_0-1/2}\bar{q}^{\bar{L}_0-1/2}(e^{-i\pi})^{\rho_\beta\cdot F}]}{Tr_\alpha[q^{L_0-1/2}\bar{q}^{\bar{L}_0-1/2}(e^{-i\pi})^{\rho_\beta\cdot F}]} \tag{2.23}$$

The last term is a total derivative and drops out after using the generalized Gauss's theo-

rem. After performing the p integration we have

$$\begin{array}{c}\mathcal{L}'(A^a_\mu) = \frac{1}{4}F^2_{\mu\nu}\frac{1}{16\pi^2}\prod_l \frac{1}{\mathcal{N}_l}\sum_{\alpha,\beta} c(\alpha,\beta)\int_\Gamma \frac{d^2\tau}{\tau_2}\int\frac{d^2\nu}{\tau_2}\langle J^a(z)J^a(1)\rangle_{\alpha,\beta} \\ \times\, 2[\langle\Psi(\bar z)\Psi(1)\rangle^2_{\alpha,\beta} - \langle X^R(\bar z)\bar z\bar\partial X^R(1)\rangle^2_{\alpha,\beta}]Tr_\alpha[q^{L'_0-1/2}\bar q^{\bar L'_0-1/2}(e^{-i\pi})^{\rho_\beta\cdot F}]\end{array} \quad . \tag{2.24}$$

The fermionic current gives rise to a gauge independent(apart from k_i) part and a gauge dependent part:

$$\langle J^a(z)J^a(1)\rangle_{\alpha,\beta} = -k_a(z\frac{\partial}{\partial z})^2\log\theta_1(z,q) + \langle J^a_0 J^a_0\rangle\,. \tag{2.25}$$

Since k_a is one, the first part will shift all the groups by the same amount and can thus be absorbed into a redefinition of the string coupling constant. Using the explicit expressions for the currents, their correlation functions are found to be:

$$\langle J^a_0 J^a_0\rangle_{\alpha,\beta} = \frac{1}{2}f^a_{cd}f^a_{cd}2q\log\theta\begin{bmatrix}\rho^c_\alpha\\ \rho^c_\beta\end{bmatrix}(0|q) \tag{2.26}$$

for the SU(2) currents(no sum on a) and

$$\langle J^a_0 J^a_0\rangle_{\alpha,\beta} = 2q\log\theta\begin{bmatrix}\rho^j_\alpha\\ \rho^j_\beta\end{bmatrix}(0|q) \tag{2.27}$$

for the U(1) currents($1\le j\le 5$).

In the $SU(3)\times U(1)\times U(1)$ model presented in sect. 4, the currents are given by:

$$J^a(z) = \bar J^a(z) + q^a(z)$$

$$\bar J^a(z) = -\frac{i}{2}f_{abc}b^b(z)b^c(z) \quad ; \quad q^q(z) = i\lambda^a_{ij} : f^i(z)\tilde f^j(z) : \tag{2.28}$$

$$J^{11}(z) = \frac{1}{\sqrt{3}}[: f^i(z)\tilde f^i(z) : +3\nu_2] \quad ; \quad J^{12}(z) = [: f^1(z)\tilde f^1(z) : +\nu_1]$$

where $3 \leq a, b, c \leq 10$, $2 \leq i, j \leq 4$, and the strucure constants of SU(3) are normalized so $C_\psi = 2$.

The correlation function of the $SU(3)$ current is(no sum on a):

$$\langle J_0^a J_0^a \rangle_{\alpha,\beta} = \frac{1}{2} f_{cd}^a f_{cd}^a 2q \log \theta \begin{bmatrix} \rho_\alpha^c \\ \rho_\beta^c \end{bmatrix} (0|q) + \lambda_{ij}^a \lambda_{ij}^{a\,*} 2q \log \theta \begin{bmatrix} \rho_\alpha^j \\ \rho_\beta^j \end{bmatrix} (0|q)\,. \tag{2.29}$$

Performing the ν integration on the gauge dependent part,we have:

$$\mathcal{L}'(A_\mu^a) = \tfrac{1}{4} F_{\mu\nu}^2 \frac{1}{16\pi^2} \prod_l \frac{1}{\mathcal{N}_l} \sum_{\alpha,\beta} c(\alpha,\beta) \int_\Gamma \frac{d^2\tau}{\tau_2} 2\langle J_0^a J_0^a \rangle$$
$$\times\, 2\bar{q} \frac{d}{d\bar{q}} \log \left(\frac{\left(\bar{\vartheta} \begin{bmatrix} \bar{\rho}_\alpha^1 \\ \bar{\rho}_\beta^1 \end{bmatrix} (\bar{q}) \right)^{\frac{1}{2}} \left(\bar{\vartheta} \begin{bmatrix} \bar{\rho}_\alpha^2 \\ \bar{\rho}_\beta^2 \end{bmatrix} (\bar{q}) \right)^{\frac{1}{2}}}{\eta(\bar{q})} \right) Tr_\alpha [q^{L_0' - \frac{1}{2}} \bar{q}^{\bar{L}_0' - \frac{1}{2}} (e^{-i\pi})^{\rho_\beta \cdot F}] \tag{2.30}$$

Now using the expression for the partition function for twisted fermions (2.4), we see that the one-loop two point background gauge field contribution to the effective lagrangian is

$$\mathcal{L}'(A_\mu^a) = -\tfrac{1}{4} F_{\mu\nu}^2 \frac{1}{16\pi^2} \int_\Gamma \frac{d^2\tau}{\tau_2} [2B_a(q,\bar{q}) + Y'] \tag{2.31}$$

where Y' is the gauge independent part and

$$B_a(q,\bar{q}) = -\prod_l \frac{1}{\mathcal{N}_l} \sum_{\alpha,\beta} c(\alpha,\beta) \frac{|\eta(q)|^{-22}}{\eta(q)} 2\bar{q} \frac{d}{d\bar{q}} \left(\left(\bar{\vartheta} \begin{bmatrix} \bar{\rho}_\alpha^1 \\ \bar{\rho}_\beta^1 \end{bmatrix} (\bar{q}) \bar{\vartheta} \begin{bmatrix} \bar{\rho}_\alpha^2 \\ \bar{\rho}_\beta^2 \end{bmatrix} (\bar{q}) \right)^{\frac{1}{2}} \Big/ \eta(\bar{q}) \right) \langle J_0^a J_0^a \rangle$$
$$\times \prod_{j=1}^{n} \left(\vartheta \begin{bmatrix} \rho_\alpha^j \\ \rho_\beta^j \end{bmatrix} (q) \right)^{1/2} \prod_{j=3}^{n\prime} \left(\bar{\vartheta} \begin{bmatrix} \bar{\rho}_\alpha^j \\ \bar{\rho}_\beta^j \end{bmatrix} (\bar{q}) \right)^{1/2} \prod_{j=1}^{m} \vartheta \begin{bmatrix} \rho_\alpha^j \\ \rho_\beta^j \end{bmatrix} (q) \prod_{j=1}^{m\prime} \bar{\vartheta} \begin{bmatrix} \bar{\rho}_\alpha^j \\ \bar{\rho}_\beta^j \end{bmatrix} (\bar{q}) \tag{2.32}$$

The first order correction to the field theory coupling is given by the coefficient of $-\frac{1}{4} F_{\mu\nu}^2$ in the one-loop two point background gauge field amplitude (2.31). In this analysis,

we use the type II string normalization of $C_\psi = 2$. To compare this result with field theory, which uses a normalization of $C_\psi = 1$, we must multiply this result by a factor $\Psi^2_{FT}/\Psi^2_{str} = x_a/2$. Here, Ψ^2 is the length squared of the longest root in the subgroup and x_a is the level of the Kac-Moody algebra. Then, following ref.[5], we get an equation for the $\overline{DR}$ couplings

$$\frac{1}{\alpha_i(\mu)} = \frac{x_a/2}{\alpha_{\rm GUT}} - \frac{b_a}{2\pi}\log\mu/M_{\rm str} + \frac{\Delta_a}{4\pi}\,. \tag{2.33}$$

The gauge independent part has been absorbed into a redefinition of the string coupling by

$$\frac{1}{\alpha_{\rm GUT}} = \frac{4\pi}{g^2_{str}} + \frac{Y}{4\pi} \tag{2.34}$$

where $Y = \int_\Gamma \frac{d^2\tau}{\tau_2} Y'$ and $\alpha_a = g_a^2/4\pi$. The massive string contributions are given by the thresholds Δ_a:

$$\Delta_a = \int_\Gamma \frac{d^2\tau}{\tau_2}[x_a B_a(q,\bar{q}) - b_a]\,. \tag{2.35}$$

The b_a are the field theory β functions given by[22]

$$b_a = -\frac{11}{3}Tr_{\rm V}(Q_i^2) + \frac{2}{3}Tr_{\rm F}(Q_i^2) + \frac{1}{6}Tr_{\rm S}(Q_i^2) \tag{2.36}$$

Here, the traces are over two-component fermions and real scalars(in field theory normalization). The massless contribution from the string is given by

$$b_a = \lim_{q\to 0} x_a B_a(q,\bar{q}) \tag{2.37}$$

and should equal the field theory result. This provides a consistency check for the string calculation. One can facilitate the numerical integration by making a change of variables:

$\tau_2 \equiv Im\tau = 1/\tau_2'$. Since $B_a(-\tau_1,\tau_2) = B(\tau_1,\tau_2)^*$, the imaginary part drops out leaving:

$$\Delta_a = -2Re\int_0^{.5} d\tau_1 \int_{1/\sqrt{1-\tau_1^2}}^{0} \frac{d\tau_2'}{\tau_2'}\left[x_a B_a(\tau_1,\tau_2') - b_a\right]. \tag{2.38}$$

3. Threshold calculation for two N=1 $SU(2)\times U(1)^5$ chiral models

We now calculate the threshold corrections for two twisted models with $SU(2)\times U(1)^5$ symmetry. In both cases, the thresholds increase the unification scale by a very small amount.

Example 1

A model with $SU(2)\times U(1)^5$ gauge symmetry [23,19] can be described by three generators $b_0, b_1, b_2 : \mathcal{N}_0 = \mathcal{N}_1 = 2 : \mathcal{N}_2$=4: K=2. The sixteen sectors of the model can be determined from the vectors ρ_{b_i} describing the generators:

$$\begin{aligned}
\rho_{b_0} &= ((1)^{12};(1)^4;(1)^{12};(1)^4) \\
\rho_{b_1} &= ((0)^{12};(0)^4;(1)^4(0)^8;(0)^2(1)^2) \\
\rho_{b_2} &= ((0)^{10}(1)^2;(1/2)^4;(0)^2(1)^2(0)^4(1)^4;(1/2)^4)
\end{aligned} \tag{3.1}$$

Figure 1 illustrates the boundary conditions for the sectors of the model. The massless states satisfy the following criteria:

(1): For the states to survive the projections, we must have:

$$e^{-i\pi\rho_{b_i}\cdot F}\alpha = \epsilon(\alpha, b_i)^*\alpha \tag{3.2}$$

(2): The left and right movers must each have a mass eigenvalue of 0:

$$\alpha' {m_L}^2 |S>= (L_0^L - 1/2)|S>= 0 \quad ; \quad \alpha' {m_R}^2 |S>= (L_0^R - 1/2)|S>= 0 \qquad (3.3)$$

The massless states come from the sectors: b_1, b_2,mm b_1b_2, b_2^2, $b_1b_2^2$, b_2^3, $b_1b_2^3$. For $g = SU(2)\times U(1)^5$, they are:

1) from the untwisted sector,

spin $(\pm 2, \pm\frac{3}{2})$	in (1;0,0,0,0,0) of g
$(\pm 1, \pm\frac{1}{2})$	in adjoint of g
$(\frac{1}{2}, 0)$	in $(1; \pm 1, 0, 0, 0, 0) \oplus 2(1; 0, \underline{1, 0, 0, 0})$ of g
$(-\frac{1}{2}, 0)$	in $(1; \pm 1, 0, 0, 0, 0) \oplus 2(1; 0, \underline{-1, 0, 0, 0})$ of g
$(\pm\frac{1}{2}, 2(0))$	in $(1; 0, 0, 0, 0, 0)$ of g

2) from the singly twisted sector,

$(\frac{1}{2}, 0)$ in $2(1; \pm\frac{1}{2}, \underline{-\frac{3}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}})$ of g

3) from the anti-twisted sector,

$(-\frac{1}{2}, 0)$ in $2(1; \pm\frac{1}{2}, \underline{\frac{3}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}})$ of g

4) from the doubly twisted sector,

$(\frac{1}{2}, 0)$ in $(1; 0, \underline{\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}}) \oplus (1; 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$

$\oplus (1; 0, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}) \oplus 2(1; 0, \underline{\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}})$ of g

$(-\frac{1}{2}, 0)$ in $(1; 0, \underline{-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{1}{2}}) \oplus (1; 0, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2})$

$\oplus (1; 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}) \oplus 2(1; 0, \underline{-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}})$ of g.

Example 2

Example 2 describes an N=1 $SU(2) \times U(1)^5$ model. The generators are :

$$
\begin{aligned}
\rho_{b_0} &= ((1)^{10}; (1)^2(1)^2(1)^6; (1)^4(1)^4; (1)^4(1)^4) \\
\rho_{b_1} &= ((0)^{10}; (0)^2(0)^2(0)^6; (1)^2(1)^2(0)^4(0)^4; (0)^4(0)^4) \\
\rho_{b_2} &= ((0)^{10}; (3/4)^2(1/2)^2(1/4)^6; (0)^2(1)^2(0)^4(1)^4; (1/2)^4(1/2)^4)
\end{aligned}
\tag{3.4}
$$

Figure 2 represents the boundary conditions on the fermions in each sector. The massless spectrum is :

Sectors b_1, ϕ:

spin $(\pm 2, \pm\frac{3}{2})$ in (1;0,0,0,0,0) of g

$(\pm 1, \pm\frac{1}{2})$ in adjoint of g

$(\pm\frac{1}{2}, 2(0))$ in (1;0,0,0,0,0) of g

$(0, \frac{1}{2})$ in 2(1;1,0,0,0,0)

$(0, -1/2)$ in 2(1;-1,0,0,0,0)

Sectors $b_2, b_2 b_1, b_2^7, b_2^7 b_1$

spin $(0, \frac{1}{2})$ in $2(1; -\frac{3}{4}, -\frac{5}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}) \oplus 2(1; \frac{1}{4}, \frac{3}{8} \underline{-\frac{7}{8}, \frac{1}{8}, \frac{1}{8}})$

$(0, -\frac{1}{2})$ in $2(1; \frac{3}{4}, \frac{5}{8}, -\frac{1}{8}, -\frac{1}{8}, -\frac{1}{8}) \oplus 2(1; -\frac{1}{4}, -\frac{3}{8}, \underline{\frac{7}{8}, -\frac{1}{8}, -\frac{1}{8}})$

Sectors $b_2^3, b_2^3 b_1, b_2^5, b_2^5 b_1$

spin $(0, \frac{1}{2})$ in $2(1; -\frac{3}{4}, -\frac{1}{8}, -\frac{3}{8}, -\frac{3}{8}, -\frac{3}{8}) \oplus 2(1 : \frac{1}{4}, -\frac{1}{8}, \underline{\frac{5}{8}, \frac{5}{8}, -\frac{3}{8}})$

$(0, -\frac{1}{2})$ in $2(1; \frac{3}{4}, \frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{3}{8}) \oplus 2(1; -\frac{1}{4}, \frac{1}{8}, \underline{-\frac{5}{8}, -\frac{5}{8}, \frac{3}{8}})$

Sectors $b_2^4, b_2^4 b_1$

spin $(2(0),\pm\frac{1}{2})$ in $(1;0,\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2})\oplus(1;0,-\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2})$

$(0,\frac{1}{2})$ in $2(1;0,-\frac{1}{2},\underline{\frac{1}{2},-\frac{1}{2},-\frac{1}{2}})$

$(0,-\frac{1}{2})$ in $2(1;0,\frac{1}{2},\underline{-\frac{1}{2},\frac{1}{2},\frac{1}{2}})$

Sectors $b_2^2, b_2^2 b_1, b_2^6, b_2^6 b_1$

spin $(0,\frac{1}{2})$ in $(1;-\frac{1}{2},\frac{3}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4})\oplus 2(1;\frac{1}{2},\frac{3}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4})$

$$\oplus(1;-\frac{1}{2},-\frac{1}{4},\underline{-\frac{3}{4},\frac{1}{4},\frac{1}{4}})$$

$(0,-\frac{1}{2})$ in $(1;-\frac{1}{2},\frac{3}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4})\oplus 2(1;\frac{1}{2},\frac{1}{4},\underline{-\frac{3}{4},\frac{1}{4},\frac{1}{4}})$

$$\oplus(1;-\frac{1}{2},-\frac{1}{4},\underline{-\frac{3}{4},\frac{1}{4},\frac{1}{4}})$$

$(0,\frac{1}{2})$ in $(1;\frac{1}{2},-\frac{3}{4},-\frac{1}{4},-\frac{1}{4},-\frac{1}{4})\oplus 2(1;-\frac{1}{2},\frac{1}{4},\underline{\frac{3}{4},-\frac{1}{4},-\frac{1}{4}})$

$$\oplus(1,\frac{1}{2},\frac{1}{4},\underline{\frac{3}{4},-\frac{1}{4},-\frac{1}{4}})$$

$(0,-\frac{1}{2})$ in $(1;\frac{1}{2},-\frac{3}{4},-\frac{1}{4},-\frac{1}{4},-\frac{1}{4})\oplus 2(1;-\frac{1}{2},-\frac{3}{4},-\frac{1}{4},-\frac{1}{4},-\frac{1}{4})$

$$\oplus(1;\frac{1}{2},\frac{1}{4},\underline{\frac{3}{4},-\frac{1}{4},-\frac{1}{4}})$$

Equivalence of the two $SU(2)\times U(1)^5$ models

The quantum numbers of each state are described by a six component vector in an $SU(2)\times U(1)^5$ space. The equivalence of the massless spectra is most easily shown by determining the length and relative orientation of the vectors in their respective spaces. Since all the states are $SU(2)$ singlets, we only need to look at the $U(1)$ quantum numbers. Comparing the two $U(1)^5$ spaces, one finds that every non-zero vector has a magnitude of one. There are also an equal number of pairs of vectors having the same angular separation in both spaces. Thus we see that they are related by a rotation of the coordinates which

describe the quantum numbers of the massless states. To be more specific, the rotations that take the first model into the second model are described by a 5x5 transformation matrix. If S_1 is a state in the first model :

$$S_1 = \begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{pmatrix} \tag{3.5}$$

a state in the second model S_2 is given by:

$$S_2 = MS_1 \tag{3.6}$$

where M is the transformation matrix given by:

$$M = \frac{1}{2} \begin{pmatrix} 0 & 2 & 0 & 0 & 0 \\ -1 & 0 & -1 & -1 & -1 \\ 1 & 0 & -1 & 1 & -1 \\ 1 & 0 & -1 & -1 & 1 \\ 1 & 0 & 1 & -1 & -1 \end{pmatrix} . \tag{3.7}$$

To compare the massive spectra, we calculate the partition function for both models. Although the partition function can be shown to be identically zero for both models, one can look at the number of massive states at each level for bosons and fermions separately. For supersymmetric theories, these contributions cancel. When we expand the partition function we obtain for the bosons of both models:

$$Z = 104 + 24\bar{q}^{\frac{1}{2}} q^{-\frac{1}{2}} + 256\bar{q}^{-\frac{1}{4}} q^{\frac{3}{4}} + 1536 q^{\frac{1}{4}} \bar{q}^{\frac{1}{4}} + ... \tag{3.8}$$

Both models have the same massless spectrum and appear to have the same number of massive states. This suggests that they are equivalent.

Thresholds

For example 1, the levels of the gauge groups are given by x_a = (2;2;2,2,2,2). Using these values, we find that the zero-mode contribution of the string (2.37) agrees with the field theory (2.36). These β-functions are given by b_a = (-6;6;9,9,9,9) for the five gauge groups. The presence of b_a ensures that the integral in (2.38) converges. A mathematica computer program was developed to calculate the expression (2.38) and to perform the numerical integrations. Each sector was evaluated independently to check for consistency with field theory. As a result we find that the thresholds for example 1 are Δ_a = (-1.25; 0.074, 0.41, 0.41, 0.41, 0.41).

We would now like to know how the massive modes of the string will change the unification scale. For this particular chiral model, it appears that there is a small increase in the unification scale. If we include the massive string contributions, the equations describing the running of the field theory coupling constants are given by:

$$\begin{aligned} \frac{1}{\alpha_2(\mu)} &= \frac{1}{\alpha_G} + \frac{3}{\pi}\log(\frac{\mu}{M_{st}}) - \frac{1.25}{4\pi} \\ \frac{1}{\alpha_{1a}(\mu)} &= \frac{1}{\alpha_G} - \frac{3}{\pi}\log(\frac{\mu}{M_{st}}) + \frac{.074}{4\pi} \\ \frac{1}{\alpha_{1b}(\mu)} &= \frac{1}{\alpha_G} - \frac{9}{2\pi}\log(\frac{\mu}{M_{st}}) + \frac{.41}{4\pi} \end{aligned} \tag{3.9}$$

Solving the first two equations in (3.9), one finds the scale at which the $SU(2)$ and first $U(1)$ meet;

$$\frac{M_u}{M_{st}} = EXP\left[\frac{(\Delta_{1a} - \Delta_2)}{2(b_{1a} - b_2)}\right] = 1.0567 \tag{3.10}$$

Similarly for $SU(2)$ and the other four $U(1)'$s, we get $\frac{M_U}{M_{st}} = 1.0568$. The two independent $U(1)'$s meet at $\frac{M_U}{M_{st}} = 1.057$.

For example 2, which has the same particle content as example 1, we get slightly different results. The β-functions are $b_a = (-6, 27/4, 9, 35/4, 35/4, 35/4)$, and the thresholds are $\Delta_a = (-1.25; 0.015, 0.408, 0.319, 0.319, 0.319)$. Comparing the results for the two models, we note that the $SU(2)$ β-functions and thresholds are the same, but the $U(1)$ β-functions and thresholds are different. This is not a surprising result. One would expect different values for the $U(1)$ β-functions and thresholds because the charges are different. Here again, we see that the threshold corrections lead to a small increase in the unification scale, which is defined as the energy at which the couplings intersect. Table 1 shows the energy (M_u/M_{str}) at which the couplings intersect.

$-$	$SU(2)$	$1st\, U(1)$	$2nd\, U(1)$	$3rd, 4th, 5th\, U(1)$
$SU(2)$	$-$	1.051	1.057	1.055
$1st\, U(1)$	1.051	$-$	1.091	1.078
$2nd\, U(1)$	1.057	1.091	$-$	1.19

Table 1- Change in unification (M_u/M_{str})

Although both models have the same particle content, they have different boundary conditions on the internal fermions. For example, the second model contains ν values like 1/8 etc.. They also have different β functions and different threshold corrections. This shows that it is possible to obtain two N=1 supersymmetric models with the same massless spectrum but different boundary conditions on the internal fermions. This feature allows one to modify the coupling without changing the particle spectrum.

4. Enhancement of gauge symmetry and mixing angle

In the standard model, the Weinberg angle arises from a rotation of the third component of SU(2) with the U(1) generator. An analogous scenario can be found with the group $SU(3) \times U(1) \times U(1)$. In this case one rotates the two U(1) charges. In the model discussed here, we also get an enhancement of the $SU(3) \times U(1) \times U(1)$ gauge symmetry to $SU(3) \times SU(2) \times U(1)$. This arises from the twisted fermions in the theory. Where we would normally find ten gauge bosons in an $SU(3) \times U(1) \times U(1)$ theory, we find twelve; two additional spin(± 1) particles coming from the twisted sectors. A rotation of the $U(1)$ generators will give these states quantum numbers similar to the W bosons of the standard model.

First, we give the Kac-Moody algebra for $SU(3) \times U(1) \times U(1)$ [24].

$$
\begin{gathered}
J^a(z) = \bar{J}^a(z) + q^a(z) \\
\bar{J}^a(z) = -\frac{i}{2} f_{abc} b^b(z) b^c(z) \quad ; \quad q^q(z) = i\lambda^a_{ij} : f^i(z)\tilde{f}^j(z) : \\
J^{11}(z) = \frac{1}{\sqrt{3}}[: f^i(z)\tilde{f}^i(z) : +3\nu_2] \quad ; \quad J^{12}(z) = [: f^1(z)\tilde{f}^1(z) : +\nu_1]
\end{gathered}
\tag{4.1}
$$

where the structure constants of SU(3) are normalized as $f_{abc}f_{abe} = \delta_{ce}\bar{C}_\psi$,

$3 \leq a, b, c \leq 10$, and $2 \leq i, j \leq 4$. λ^a_{ij} is an antihermitian representation of SU(3):

$$
\lambda^{a*}_{ij} = -\lambda^a_{ji} \quad ; \quad [\lambda^a, \lambda^b] = f_{abc}\lambda^c \quad ; \quad Tr\lambda^c\lambda^e = -\delta_{ce}\frac{C^{(q)}_\psi}{2} \tag{4.2}
$$

where

$$
C_\psi \equiv \bar{C}_\psi + C^{(q)}_\psi \quad ; \quad \frac{C_\psi}{\bar{C}_\psi} = \frac{4}{3}\,.
$$

These equations fix the normalization of the $U(1)$ charges.

We now present a N=4 $SU(3)\times U(1)\times U(1)$ model. The number of sectors depends on the actual values for ν_1 and ν_2. We first consider a model with undetermined values of ν and derive the values that give a unitary theory and enhance the symmetry. The generators are described by three 16-dimensional vectors given in (4.3) and illustrated in Fig. 3.

$$\begin{aligned}
\rho_{b_0} &= ((1)^{12};(1)^2(1)^6;(1)^8;(1)^{12})\\
\rho_{b_1} &= ((0)^{12};(0)^2(0)^6;(1)^8;(0)^{12})\\
\rho_{b_2} &= ((0)^{12};(2\nu_1)^2(2\nu_2)^6;(0)^8(0)^{12})
\end{aligned} \tag{4.3}$$

Any superstsring theory must be unitary. It has been shown [18] that the necessary requirements for unitarity and modular invariance are:

$$\mathcal{N}_i\rho_{b_i}\cdot\rho_{b_i} \equiv 0 \bmod 8$$

for $\mathcal{N}_i$ even: (4.4)

and

$$\mathcal{N}_{ij}\rho_{b_i}\cdot\rho_{b_j} \equiv 0 \bmod 4$$

where $\mathcal{N}_{ij}$ is the least common multiple of $\mathcal{N}_i$ and $\mathcal{N}_j$.

The additional gauge bosons come from massless states where twisted fermions act on the vacuum. Since we want particles that are counterparts to the $W\pm$ bosons of the standard model, the massless states that will transform to these gauge bosons should be

singlets of $SU(3)$. This is accomplished by requiring that the massless states be excited states of the projection ν_1. The mass equation for the left movers now becomes:

$$\alpha' m_L^2 |S>= (L_0^L - 1/2)|S>= 0$$

$$0 = (\sum_{r \in Z+\lambda} r : \tilde{f}_{-r} f_r : + \frac{1}{4} 2{\nu_1}^2 + \frac{1}{4} 6{\nu_2}^2)|0> \tag{4.5}$$

In order for these states to survive the projections, (3.2) must also be satisfied. If we let $\nu_2 = \frac{l}{n}$ then the conditions above,(4.4), (4.5), and (3.2) give:

$$\nu_1 = 1 \pm \sqrt{1 - 3{\nu_2}^2} \tag{4.6}$$

and the diophantine equation:

$$n^2 - 3l^2 = m^2 \tag{4.7}$$

where n, m, l $\in Z$. Thus we find that many different boundary conditions are allowed for this particular set of generators. Now we will show that these boundary conditions give rise to different mixing angles. This is illustrated with a model with definite boundary conditions that satisfy (4.7). I choose $\nu_1 = \frac{1}{14}$ and $\nu_2 = \frac{3}{14}$. This model has 56 sectors and the massless particle spectrum is:

Sector b_1, ϕ

$\{\pm 2, 4(\pm 3/2), 6(\pm 1), 4(\pm 1/2), 2(0)\}$ in singlet

$\{\pm 1, 4(\pm 1/2), 6(0\}$ in adjoint

Sector $b_2, b_2 b_1$:

$\{\pm 1, 4(\pm 1/2), 6(0)\}$ in $(1, \frac{1}{\sqrt{3}} 9/14, -13/14)$

Sector $b_2^{13}, b_2^{13} b_1$:

$\{\pm 1, 4(\pm 1/2), 6(0)\}$ in $(1, -\frac{1}{\sqrt{3}} 9/14, 13/14)$

The last two states are the extra spin(± 1) particles. We now perform a rotation of the generators to change the quantum numbers of these states. We define linear orthogonal transformations of $U(1) \times U(1)$ as:

$$U(1)'_y = \cos\theta\, U(1)_y + \sin\theta\, U(1)_w$$

$$T_3 = U(1)'_w = -\sin\theta\, U(1)_y + cos\theta\, U(1)_w \tag{4.8}$$

and recall that in the standard model, the isospin and hypercharge are related to the electric charge as $Q = T_3 + \frac{Y}{2}$. In order to relate the generators defined above in (4.8) to the generators of the standard model, we invert the expressions in (4.8) and compare them with $Q = T_3 + \frac{Y}{2}$ and $Q' = T_3 \cot\theta - \frac{Y}{2} \tan\theta$, where Q is the generator associated with A_μ and $Q'$is the generator associated with Z_μ. Both sets of equations have the same form if we let $Q = -U(1)_y / \sin\theta$, and $Y/2 = -\cot\theta U(1)'_y$.

We determine $\sin\theta$, $\cos\theta$ in (4.8) by requiring that the quantum numbers of the massless states with spin (± 1) have similar quantum numbers to the gauge bosons of the standard model. In Table 2, it is shown how the states of this model should transform so as to have quantum numbers like the Z,W±, and photon of the standard model.

state	$SU(3)\times U(1)_y \times U(1)_w$	$SU(3)\times U(1)'_y \times U(1)'_w$
$\tilde{f}_{-\frac{3}{7}}\vert 0>_L \otimes \epsilon \cdot b_{-\frac{1}{2}}\vert 0>_R$	$(1:\frac{1}{\sqrt{3}}(\frac{9}{14}):-\frac{13}{14})$	$(1;0;-1)$
$f_{-\frac{3}{7}}\vert 0>_L \otimes \epsilon \cdot b_{-\frac{1}{2}}\vert 0>_R$	$(1;-\frac{1}{\sqrt{3}}(\frac{9}{14});\frac{13}{14})$	$(1;0;1)$
$b^{11}_{-\frac{1}{2}}\vert 0>_L \otimes \epsilon \cdot b_{-\frac{1}{2}}\vert 0>_R$	$(1;0;0)$	$(1;0,0)$
$b^{12}_{-\frac{1}{2}}\vert 0>_L \otimes \epsilon \cdot b_{-\frac{1}{2}}\vert 0>_R$	$(1;0;0)$	$(1;0;0)$

Table 2- Quantum numbers of gauge bosons

For a general model with boundary conditions ν_1 and ν_2, there are also states like that shown in Table 2. For the general case, the quantum numbers can be written in terms of the boundary conditions ν_1 and ν_2. For a state like the first state in table 2, the $U(1)$ quantum numbers are $U(1)_y = \sqrt{3}\nu_2$, and $U(1)_w = -1+\nu_1$. It is now possible to determine the rotation required by solving (4.8) for a state with these quantum numbers.

$$\sqrt{3}\nu_2 \cos\theta + (-1+\nu_1)\sin\theta = 0$$

$$-\sqrt{3}\nu_2 \sin\theta + (-1+\nu_1)\cos\theta = -1 \tag{4.9}$$

Solving this for $\cos\theta$ and $\sin\theta$, one obtains:

$$\cos\theta = -(-1+\nu_1)$$

$$\sin\theta = \sqrt{3}\,\nu_2 \tag{4.10}$$

$\sin\theta$ is analogous to the Weinberg angle.

In the N=4 supersymmetric model presented above, the symmetry group of the standard model arises by an enhancement of the gauge symmetry $SU(3)\times U(1)\times U(1)$. This

enhancement was brought about by the complex twisted fermions of the model. One can obtain gauge bosons in adjoint representations by making a rotation of the gauge generators as in (4.8). This gives rise to a mixing angle analogous to the Weinberg angle of the standard model. It has been shown that this mixing angle depends on the boundary conditions of the internal fermions as in (4.10). Once this has been determined for a realistic string model, the β -function and threshold corrections would allow one to calculate the couplings at measurable energies. Without further restrictions, one could choose different boundary conditions to give an acceptable mixing angle.

5. Conclusion

This paper is concerned with the calculation of threshold corrections in twisted string models and how they might change the unification point. For the two N=1 $SU(2)\times U(1)^5$ models presented, the threshold corrections lead to a very small increase in the unification scale. In the examples chosen here, all moduli are fixed to be at the point where the gauge symmetry is enlarged. So, unlike heterotic models, there are no free parameters which can be adjusted to give large thresholds.

As a further step in making a connection with low energy phenomenology, we showed how the complex twisted fermions can enhance the symmetry of an N=4 $SU(3)\times U(1)\times U(1)$ model to the gauge group $SU(3)\times SU(2)\times U(1)$. The boundary conditions on the twisted fermions of this model give massless spin ± 1 particles that are singlets of $SU(3)$ and a model that is unitary and modular invariant. We find, for this particular model at least, that there are many possible boundary conditions that would have the same

particle spectrum. When these massless spin± 1 particles are required to have quantum numbers like the gauge bosons of the standard model, a mixing angle analogous to the Weinberg angle can be identified. An expression can then be derived for the dependence of this mixing angle on the internal boundary conditions of the model. Once more realistic string models are developed, the methods used here should allow one to compare the string theory with measurable quantities. Using the β- function and string thresholds, one could determine the gauge couplings at low energies. If there are no additional constraints on the boundary conditions of the internal fermions, one would have the freedom to choose a model that best describes the low energy physics.

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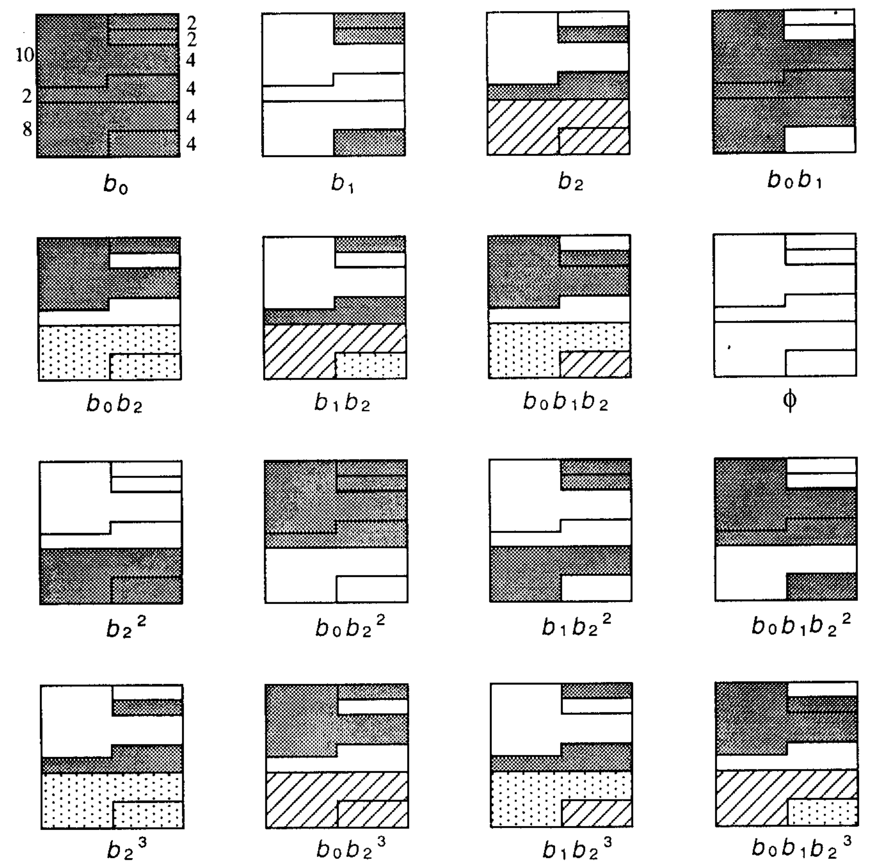


FIG. 1. Boundary conditions for the world sheet fermions of the $N = 1$ SU(2) $\times$ U(1)^5 model of example 1.

The boundary conditions of the internal fermions are given by the following four shadings:

R ($\nu = 1/4$), NS ($\nu = 0$),

($\nu = 1/4$), ($\nu = -1/4$),

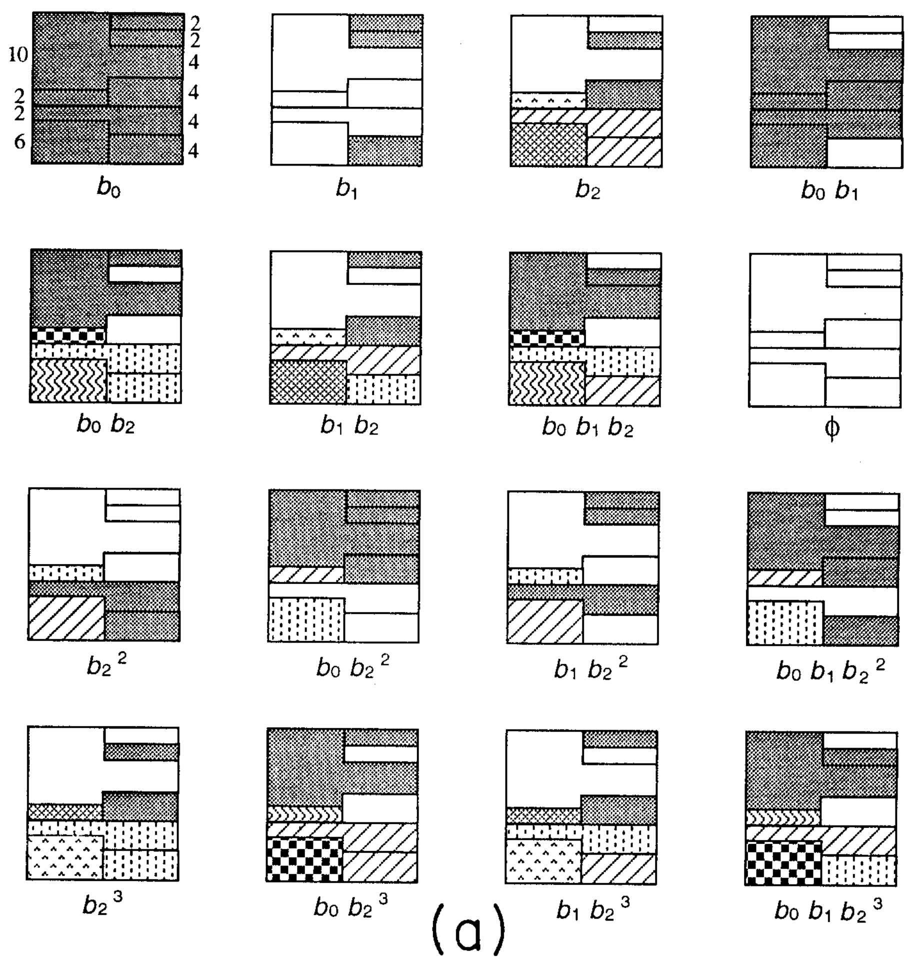
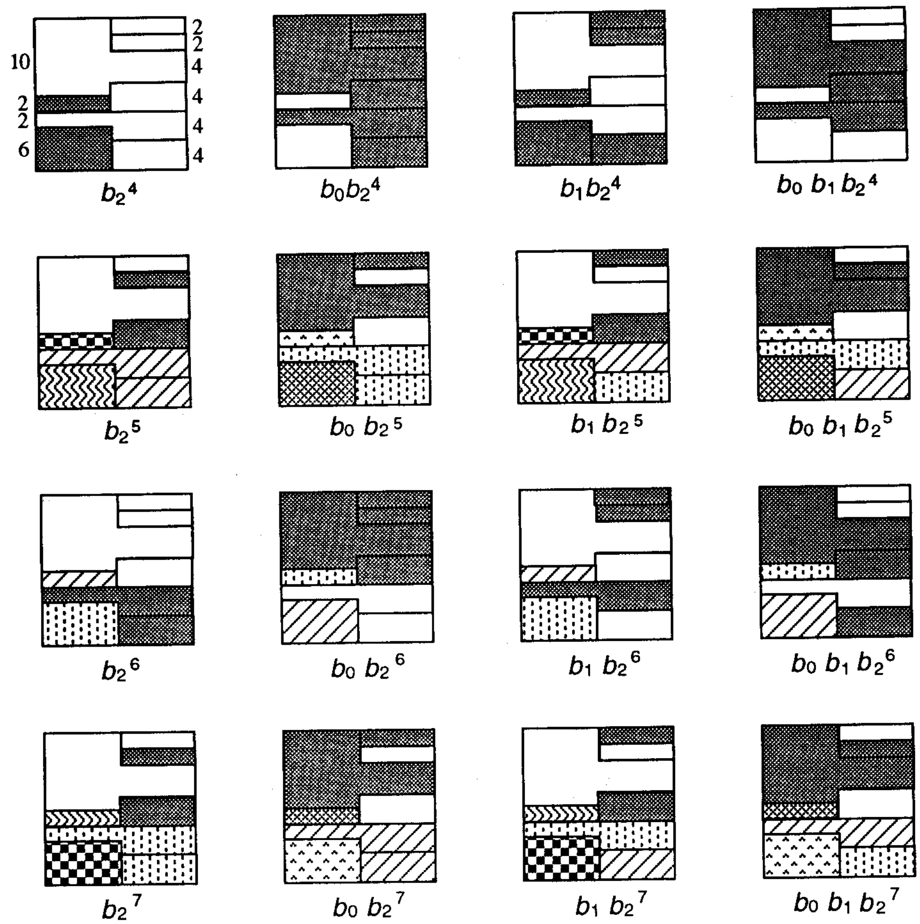


FIG. 2. Boundary conditions for the world sheet fermions of the $N = 1$ SU(2) $\times$ U(1)^5 model for example 2. This model has the same particle spectrum as the model of example one, but it has different boundary conditions.



The boundary conditions of the internal fermions are given by the following shadings:

R (ν = 1/2), NS (ν = 0), (ν = 1/8), (ν = 3/8),

(ν = 1/4), (ν = -1/4), (ν = -1/8), (ν = -3/8),

(b)

FIG. 2 *(Continued)*.

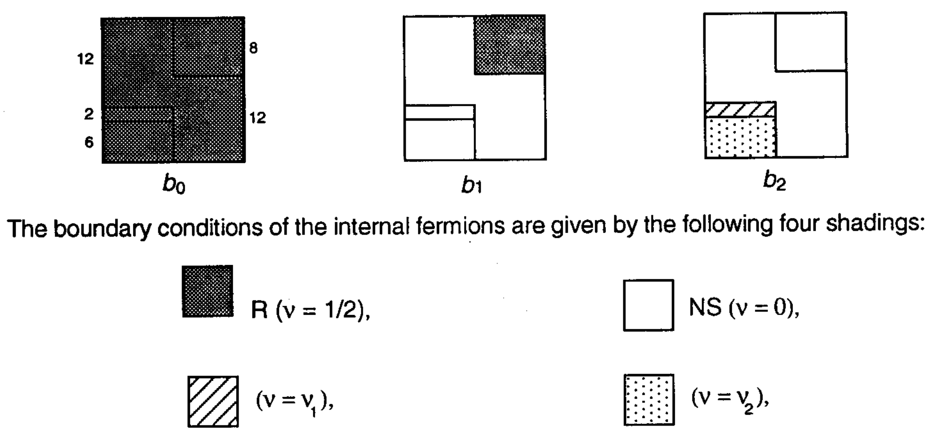


FIG. 3. Boundary conditions for the world sheet fermions of the generator sectors of the $N = 4$ SU(3) × U(1) × U(1) model of example 3. A rotation of the gauge generators enhances the symmetry group.